Reply to Comment by Paul R. Motyka and G. Warren Hall

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WE wish to thank P. R. Motyka and G. W. Hall for their comments[1] on our decoupling paper.[2] They have correctly explained why we arrived at a result different from theirs when solving their problem.[3] When the calculations presented in Ref. 2 were done, we were at a loss to explain the nonagreement of results. It was discovered that their solution did not give the prescribed eigenvalues and we incorrectly assumed that was the explanation. As Motyka and Hall[1] have correctly pointed out, we had, in fact, assigned the eigenvalues to different eigenvectors.

It is not true, however, that this is an inherent shortcoming of the geometric approach. As was pointed out,[2] the geometric theory provides a method for placing the poles that was not used in Ref. 2. In fact, for the lateral motions case, it is a simple matter to assign eigenvalues independently to the individual controllability subspaces.[4] It is true that the heuristic Penalty Function Method described in Ref. 2 must be used with considerable care, and we are indebted to Motyka and Hall for making this point.

Next, we consider the comments relating to "exact" model following. In order to minimize confusion, we shall try to be somewhat precise. In the notation of Ref. 3, the given system is described by

$$\dot{x} = Fx + Gu \tag{1}$$

while the model obeys

$$\dot{x}_m = F_m x_m + G_m u_m \tag{2}$$

Since it is desired that the system behave like the model, we equate the right sides of Eq. (1) and (2), put $x_m = x$, and for fixed x and u_m, "solve" for u. Hall and Motyka[3] give

$$u = (G^T G)^{-1} G^T [(F_m - F)x + G_m u_m]$$

as the "solution." As pointed out in Ref. 2, such u minimizes‡ $||\dot{x} - \dot{x}_m||$, but need not make the difference zero. If one substitutes the above value of u into Eq. (1), it can be shown that

$$\dot{x} - \dot{x}_m = [I - G(G^T G)^{-1} G^T][(F_m - F)x + G_m u_m].$$

For the general structure of G, given in Ref. 3, it is found that the first term in the above product is the matrix $[\theta\ e_2\ \theta\ \theta]$ (where $e_2 = [0\ 1\ 0\ 0]^T$ and $\theta = [0\ 0\ 0\ 0]^T$). It is clear then that $\dot{x} = \dot{x}_m$ only if the second row of $[(F_m - F)x + G_m u_m]$ is zero. Since the model has $\phi_m = p_m$, it is necessary that $\phi = p$. Of course, if $\phi \neq p$, it is possible to hypothesize a model for which $\dot{x} = \dot{x}_m$. In this sense, it was, perhaps, a bit strong to say that the method fails. However, it is hoped that the above discussion clarifies our point. More complete discussions of the implicit model following method may be found in Refs. 6 and 7.

Finally, in order that the reader may be able to judge the relative merits of the various approaches to decoupling, we point out that for many problems it is not possible to decouple and stabilize a system using state feedback only. It has been shown[8] that if one attempts to decouple speed and flight path angle in the longitudinal motions of an aircraft using state feedback alone, then except in contrived cases, there will be a pair of fixed eigenvalues. In order to alleviate this difficulty, one must use dynamics in the feedback controller. This problem is treated in a natural way in the geometric theory,[9,10] whereas we suspect it would be generally difficult to formulate and solve by model following methods.

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‡Under the assumption that G has full rank, the quantity $(G^T G)^{-1} G^T$ is the *generalized inverse* of G[5].

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Comments on "Low-Area Ratio, Thrust-Augmenting Ejectors"

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FANCHER'S paper[1] contains errors and omissions which render his conclusions rather ambiguous, at best. The following observations are offered in support of this contention. Fancher's notation is retained as far as possible, and new equations which parallel those in Ref. 1 are indicated by means of barred numbers.

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Ideal Ejector with Diffusion

Figure 2 of Ref. 1 presents theoretical augmentation ratio for an ideal ejector (no nozzle, inlet, diffuser, or mixing duct wall friction losses) as a function of inlet area ratio for several diffuser area ratios. Unfortunately, the theoretical development which precedes Fig. 2 is valid only for the zero-diffusion case (constant-area ejector). Fancher does not explain how the augmentation values in Fig. 2 for $A_3/A_2 > 1.0$ were arrived at, and a numerical spot check indicates that they are in error.

The mixed flow exits from the ejector in general at the end of the diffuser, station 3, rather than at the end of the mixing duct, station 2. The augmentation ratio is therefore $\phi = \dot{m}_3 V_3/\dot{m}_0 V_0'$ which, using continuity for the diffuser, may be rewritten as:

$$\phi = (A_2/A_3)(\dot{m}_2/\dot{m}_0)(V_2/V_0)V_0/V_0' \qquad (\bar{1})$$

Comparison of Eq. ($\bar{1}$) with Eq. (1) of Ref. 1 reveals that the latter omits the factor A_2/A_3, and is therefore valid only when this factor is unity.

Equations (2) and (3) of Ref. 1 are correct, since they pertain only to the flow in the mixing duct even in the case with diffusion. Eq. (4), however, is incorrect: the inlet velocity ratio is altered by diffusion for any given inlet area ratio, and this diffusion dependence is absent. Defining

$$y \equiv (A_2/A_3)^2 - 1$$

it can be shown that the inlet velocity ratio in the presence of diffusion is given by:

$$u = \frac{V_1}{V_0} = \frac{[2x + (2x-1)y]^{1/2}(1+x) - (2+y)x}{1 + (1+y)x^2} \qquad (\bar{4})$$

When $A_2/A_3 = 1$ (no diffusion), Eq. ($\bar{4}$) reduces to Eq. (4) of Ref. 1.

The augmentation values with diffusion presented in Fig. 2 of Ref. 1 are somewhat optimistic, according to a spot check by the present writer. For the case of $A_3/A_2 = 2.50$, for example, Fig. 2 yields values of ϕ of approximately 2.75 and 3.45 at inlet area ratios of, 10 and 30, respectively; the corresponding values calculated by the present writer are 2.38 and 3.04.

It should also be noted in passing that the incompressible analysis of Ref. 1 tacitly assumes the densities of the primary and secondary flows to be equal. In practical applications, however, the primary flow is normally hotter and, therefore, lower in density than the secondary. This circumstance causes performance of even the ideal ejector to be degraded, as has been shown for the constant-area ejector by Heiser.[2]

Ejector with Flow Losses

It should be remarked that even the ideal ejector experiences an entropy rise due to mixing, and therefore is not "loss-free." This inherent inefficiency of the ejector is reflected in the ideal ejector equations. As for additional losses occasioned by wall friction and the like, it will only be pointed out here that Eq. (11) of Ref. 1 for thrust augmentation in the presence of these additional losses is in error.

First, note that for the ideal ejector one may use Eqs. (2) and (3) of Ref. 1 together with Eq. ($\bar{1}$) to obtain:

$$(A_2/A_3)(1+ux)^2/(1+x)(1-u^2)^{1/2} \qquad (\bar{1}')$$

Eq. (11) of Ref. 1 differs from Eq. ($\bar{1}'$) only by the factor η_N, the primary nozzle velocity efficiency. Note that the diffuser area ratio factor omitted from Eq. (1) of Ref. 1 is included in Eq. (11).

To derive the expression for ϕ with losses, one may define the following efficiencies for the primary nozzle and secondary inlet, respectively:

$$\bar{\eta}_N \equiv \rho(V_0')^2/2(p_{to} - p_{amb})$$

$$\bar{\eta}_I \equiv \rho V_1^2/2(p_{amb} - p_1)$$

where p_{to} is the total pressure ahead of the primary nozzle, p_{amb} is the ambient static pressure, and other quantities are as defined by Fancher. The bars signify that these definitions are introduced here rather than in Ref. 1; the relationship between the nozzle efficiency of Ref. 1 and that defined above is $\eta_N = (\bar{\eta}_N)^{1/2}$.

In this more general case with losses, Eq. (3) of Ref. 1 no longer holds. Instead, the identity

$$p_{to} - p_0 = (p_{to} - p_{amb}) + (p_{amb} - p_0)$$

together with the above efficiency definitions and the fact that $p_0 = p_1$ leads readily to

$$V_0/V_0' = [1 - (\bar{\eta}_N/\bar{\eta}_I)u^2]^{-1/2} \qquad (\bar{3})$$

In the case of the ideal ejector, $\bar{\eta}_N = \bar{\eta}_I = 1$ and Eq. ($\bar{3}$) reduces to Eq. (3) of Ref. 1.

Using Eqs. ($\bar{1}$) and ($\bar{3}$) together with Eq. (2) of Ref. 1, one finds:

$$\phi = (A_2/A_3)(1+ux)^2/(1+x)[1 - (\bar{\eta}_N/\bar{\eta}_I)u^2]^{1/2} \qquad (\overline{11})$$

It is thus seen that the nozzle efficiency does not enter as the simple factor given in Eq. (11) of Ref. 1, and that it enters in combination with the secondary inlet efficiency, the latter affecting the amount of secondary flow that is pumped.

Experimental Findings

Fancher's central thesis seems to be that the performance of low-area ratio ejectors can be markedly increased by promoting faster mixing between the primary and secondary flows, since this enables the mixing duct to be shortened and thereby leads to reduced wall friction losses. It is not clear how this is supported by anything in the experimental program reported in Ref. 1. For example, Fig. 5 gives theoretical curves showing the influence of mixing length on augmentation, but their validity is not supported by any cited experimental data. As for Fig. 9, values of L/W are not cited and there is no indication that L/W was experimentally optimized; nor is there any comparison given between the performance of the configuration in Ref. 1 and a comparable conventional ejector configuration.

It must also be pointed out that the effective area A_0 of the primary nozzle as found via indirect means by Fancher (12.6 in.^{2}) is not reasonable. Dividing this area by the total height (24.5 in.) of the two-dimensional nozzle shown in Fig. 7 gives an effective nozzle width $W_0 = 0.51$ in. For comparison, a straight line between the lips of the asymmetric nozzle should if anything overestimate W_o; yet, using Fig. 7 again, one obtains $W_o = 0.27(2)^{1/2} = 0.38$ in this manner. Therefore, all experimental inlet area ratios in Ref. 1 are evidently in error, true area ratios being higher than those calculated. Since increasing inlet area ratio tends to improve performance, this error alone may account for any seeming improvements in performance over conventional ejectors achieved by the configuration of Ref. 1.

Thus, it must finally be concluded that Eq. (12) of Ref. 1, which is intended as an approximate guide to achievable ϕ, is without visible means of support: it is based on Fig. 9, whose experimental data are apparently incorrect and whose theoretical curves are (noting the earlier analytical comments) at best inaccurate.

References

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Reply to Comment by Philip A. Graham

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GRAHAM is correct in observing that Eqs. (1) and (4) of the original paper refer only to the zero diffusion case; this was all they were intended to do, since they describe the performance of the simple ejector shown in that work. A development with diffusion was not included, as it can be found in a number of the sources referenced. Inclusion of a perfect diffuser results in Graham's Eq. (4).

Figure 2 of the original work is incorrect. In redrawing this figure from the source paper[3] an error was made. Figure 2 of the source paper gives Graham's computed values. We accept full responsibility for this error.

In the Aerospace Research Laboratories' thrust augmentation concept the source of the primary flow is envisioned as the bypass air from a turbofan engine, and so there is no significant density difference between the mixing flows. The incompressible analysis is therefore appropriate. However, it should be understood that Heiser's[2] analysis only considers the effect of the density difference. A hotter primary jet would have a lower density, but it would also be more viscous. The net effect on the level of augmentation cannot be predicted by the simple analyses presently in use.

Graham has questioned the experimental results. The basis of his objection is his belief that the nozzle was 24.5 in. long. This dimension was unfortunately added to the figure by the Journal editors. The actual nozzle length was 36 inches which could be computed from the text which states "A unique feature is the "hypermixing" primary nozzle which is segmented into 24 elements 1½ in. long." Using the 36 in. figure for this length and the 12.6 square in. for the area, one obtains $W_0 = 0.35$ inches, slightly less than 0.38 as Graham correctly implies it should be.

No reference was made to a "loss free" ejector as Graham's use of quotation marks implies. In fact a significant portion of the mixing section of Fancher's work is devoted to a discussion of the transfer efficiency associated with the mixing loss. This loss is implicitly accounted for in the definition of thrust augmentation ϕ, since the momentum flux from the ejector reflects the transfer efficiency of the mixing process.

Regarding the definition of augmentation, Graham has chosen a different reference thrust than the one used by Fancher. The augmentation can be defined in terms of the thrust of the unshrouded primary nozzle, but then ejectors with different primary nozzles cannot be directly compared. A better definition is the one used by Fancher in which the reference thrust is the value computed for an isentropic expansion of the primary mass to atmospheric pressure. Ejectors can thus be compared in terms of thrust per horsepower required. More importantly, the nozzle efficiency becomes part of the design optimization. The development of the hypermixing nozzle is a consequence of this approach, since it was felt at ARL that an improvement in the performance of short aircraft ejectors could be obtained by increasing the rate of entrainment, even at some cost in nozzle efficiency.

Graham has misunderstood the basic conclusion that can be drawn from this work. The effect of hypermixing is not to improve the performance of a given low area ratio ejector in the sense of bringing it closer to the ideal. In fact, the lower velocity efficiency of hypermixing nozzles is a penalty that must be paid for their use. What is achieved is a higher thrust augmentation in an ejector of given length, a short length that is practical for aircraft installation. This is important to understand. The best ejector performance at a given area ratio is obtained when mixing is complete. This can be achieved either with a long mixing duct, which prohibits aircraft installation, or in a lesser distance with hypermixing nozzles at some penalty in maximum augmentation.

The comparison between two ejectors of identical geometry except for the hypermixing nozzle has been performed at ARL by Bevilaqua.[4] The results of this test are summarized in the figure. The isentropic thrust of the primary flow is used to define the augmentation. It can be seen that at low diffuser area ratios, the slot nozzle produces larger values of augmentation than either of the hypermixing nozzles. However, as the diffuser angle is increased further, the diffuser in the configuration assembled with the slot nozzle stalls, and there is a resultant loss in performance. Both hypermixing nozzles give higher maximum augmentation than the slot nozzle. These results have been discussed at length in Ref. 4.

In conclusion, we would like to apologize for the error in the original article and any confusion it may have caused. We also wish to thank Mr. Graham for his comments.

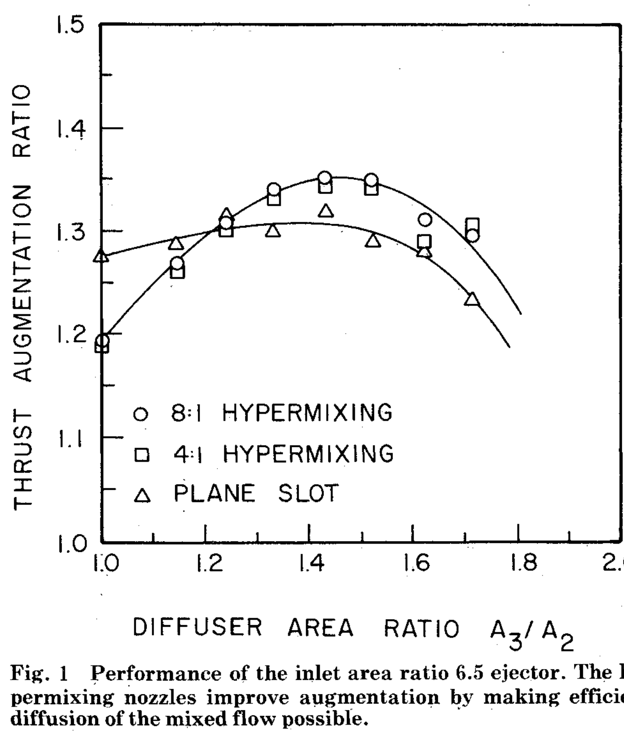

Fig. 1 Performance of the inlet area ratio 6.5 ejector. The hypermixing nozzles improve augmentation by making efficient diffusion of the mixed flow possible.

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